

Question 1] prove that the four fourth roots of unity i.e., the set $\{1, -1, i, -i\}$ is an Abelian group w.r.t. multiplication.

Proof: Suppose $M = \{1, -1, i, -i\}$.

(i) Closure Property :- closure property is satisfied as we have

<u>Closure Property</u>	
$1 \cdot 1 = 1$	$(-1) \cdot 1 = -1$
$1 \cdot (-1) = -1$	$(-1) \cdot (-1) = 1$
$1 \cdot i = i$	$(-1) \cdot i = -i$
$1 \cdot (-i) = -i$	$(-1) \cdot (-i) = i$
$i \cdot 1 = i$	$(-i) \cdot 1 = -i$
$i \cdot (-1) = -i$	$(-i) \cdot (-1) = i$
$i \cdot i = -1$	$(-i) \cdot i = 1$
$i \cdot (-i) = 1$	$(-i) \cdot (-i) = -1$

clearly every entry is $+1, -1, +i$, or $-i$.
Hence M is closed.

(ii) Associativity: Associative property is satisfied as we have

$$(1 \cdot i) \cdot (-i) = 1 \cdot \{i \cdot (-i)\} = 1,$$

$$\{1 \cdot i\} \cdot (-1) = 1, 1 \cdot \{i \cdot (-1)\} = -i \text{ etc.}$$

(iii) Existence of Identity:- Axiom on identity is satisfied, 1 being the multiplicative identity.

(iv) Existence of inverse:- Axiom on inverse is satisfied since the inverse of each element of the set exists.

$$1 \cdot 1 = e = 1$$

$$(-1) \cdot (-1) = e = 1$$

$$i \cdot (-i) = e = 1$$

$$(-i) \cdot i = e = 1$$

(v) commutativity:- The commutative law is also satisfied as

$$1 \cdot (-1) = (-1) \cdot 1,$$

$$(-1) \cdot i = i \cdot (-1) \text{ etc.}$$

Hence M is a group.

Also M is an abelian group since commutativity is also satisfied.

Hence the result

CHITRANJAN SENGUPTA

DEPTT. OF MATHEMATICS

SHERSHAH COLLEGE, SASARAM.

(KONTA)